.

OPPONENT'S REPORT

ON HABILITATION THESIS

"Kinks and oscillons: Dynamics of fields in 1+1 dimensions"

submitted by RNDr Filip Blaschke, PhD

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The first impression is that the title of the thesis should have been re-ordered: In a broader context of the study of dynamics of fields the author restricted his attention to the systems living in 1+1 dimensions. In this framework, he decided to select, for the purposes of this work, the theory and models exhibiting the emergence of the so called solitons. Finally, the scope of the thesis is still narrowed to cover the solitonic and solitons-resembling solutions of nonlinear equations known under the names of kinks and oscillons.

Needless to add that the field of such a specification is, by itself, still fairly broad. Moreover, in the author's own words, even "a unified understanding of soliton's dynamics is, despite the 40+ years of investigations, still lacking." Not too surprisingly: the subject connects several areas of a truly challenging mathematical research as well as of a multi-sided physical applicability. In this sense, the author's decision of description of the "recent advancements in understanding the two most prominent solitons in (1+1)-dimensions, namely, kinks and oscillons" is methodically well founded.

The form of the thesis is standard: The more or less self-contained text of part I (cca 100 reader-friendly explanatory pages) is followed by the set of reprints of a few selected papers in part II. It is only to be regretted that in the Table of Contents, interested reader does not find the coordinates of these reprints. Incidentally, such a list of references really exists, but it only appears, for some mysterious reasons, hidden on page 103. Moreover, what is also slightly strange is that the last two of the papers (published already in Phys. Rev. Lett. and in PTEP) are only presented in their arXived preprint form. Last but not least, the present opponent noticed the rather deplorable complete absence of any reference to the team's most recent publication in Phys. Rev. D 111, 036034. Although only published on February 26, 2025, the text would fit the thesis' volume, especially because the similar volumes are, nowadays, accessible (practically to all of us) in their easily upgraded electronic versions rather than in their traditional printed-book formats.

The style of the thesis is, as I already indicated, wide-readership-oriented, "partly conceived as an introduction in the subject". Naturally, the author's hope of his text being "self-contained" was just a dream: for the purposes of habilitation, the reasonable extent of the text is sort of first commandment. Still, it is a pity that the style of part I is, in places, over-talkative and much more informal than necessary (see, e.g., p. 42 and the sentence " However, if we stare long enough at the static equations of motion . . . we will eventually realize two things. First, . . . "). Contradicting the author's "attempt (vainly) to be self-contained." Carrying many features of a lecture for undergraduates rather than of a more compact text designed for a defense before a committee.

In this sense, the author-proposed and partially pedagogically motivated opportunity of our reading the first two chapters as an "introduction to the vast and multifaceted realm of solitons" seems to have been missed. No surprise – “mission impossible”. Those of us who do remember the real boom of the subject in the seventies are all well aware that the thesis really does not provide any space for the iconic historical comments, say, on the horse-riding John Scott Russell who observed a solitary wave in the Union Canal in Scotland in 1834, etc. Indeed, the author of the thesis is perfectly right when warning us that also the mere 61 items of the bibliography “contain only the most crucial references”, and that there exist only too many other papers which would deserve to be quoted, even just for introductory purposes.

The main scientific message is delivered in sections 3 and 4. The author reports there his passage through the “two avenues of research” representing, respectively, the study of the “oscillon-Q-ball correspondence” and of the “collective coordinate methods for understanding the kinks and their scattering”. At this point, it is probably worth adding that what is truly innovative is the coverage of the whole class of generic models. Still, close connections are preserved with the very specific “benchmark” sine-Gordon system for which the explicit constructions of the solutions of various types (viz. kinks, antikinks, breathers as well as their mutual scatterings) were always based on the existence of its exceptional symmetries and integrability features. Thus, it is worth emphasizing that a weakening of the assumptions behind the exceptional sine-Gordon model can be perceived as one of the most impressive news delivered by the present thesis.

Even having all this in mind, the current opponent re-ordered, intuitively, the reading of the text. Being addressed, first of all, by an immediate technical appeal of the innovation described in section 4 and carrying the name of “mechanization”. In a broader perception, indeed, this is a straight attempt at a deeper understanding of the structures behind dynamics of a full-fledged scalar field theory “by employing a clever ansatz.”

At the first sight, the idea (constraining, in essence, the infinite-dimensional configuration space to a finite-dimensional subspace) looks highly unorthodox and only too straightforward to be implementable. The readers of the thesis are confronted with strong statements demonstrating the opposite: The resulting reduction of the complexity of the continuous field dynamics is shown efficient. For several reasons explained by the author and "pinning down the crucial degrees of freedom" in the manner strongly inspired by Bogomol'nyi, Prasad and Sommerfeld (cf., in particular, Eq. (2.28)).

What is obtained is an algebraically tractable effective Lagrangian which is shown to yield a sufficiently reliable "picture of what is going on." Thus, although the terminology using the name of a collective coordinate model (CCM) can be considered slightly misleading, a detailed implementation of the approach is persuasive.

Paradoxically, the text of the last section 4 reflects, mainly, the contents of the first two "earlier" papers I and II. Could one infer that also the author himself inverted the ordering of the sections during a last-minute revision? In the eyes of the present opponent, the abandoned time-ordered presentation of results might make sense. Indeed, a less formal reason could be seen in the fact that the contents of papers III and IV can be read as a phenomenological climax of the story. Connecting oscillons with a (very slow) decay of various Gaussian data. Chaotic, fractal-like patterns are revealed to exist, resembling their close analogues encountered in the kink-antikink collisions, say, in the exactly solvable sine-Gordon setting. Moreover, also the mathematical aspects of the related new and numerical results are impressive, including "a renormalization group-inspired method for finding a perturbative connection between thee objects", etc.

Summarizing, the presented account of the results published in the quintuplet of the recent papers can be accepted as a habilitation thesis for defense. Marginally, let us add that there are just very few misprints and minor imperfections in the text. In the Preface (notice that the author used the occasion of switching to Latin: Praefatio), for example, only a true pedant would search for a trace of the author's hypothetical later decision of splitting the introductory text in two separate sections. Thus, "the very recent discovery about the oscillons" is outlined in the third rather than in the second chapter. Subsequently, the fourth rather than the third chapter "descibes (read: describes) the so-called mechanization."

In conclusion, the present referee feels pleased to inform the Scientific Council of the Institute of Physics of the University in Opava that the submitted habilitation thesis demonstrates, beyond any doubt, that its author, RNDr Filip Blaschke, PhD, is fully qualified for being awarded the title of "docent".

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Reviewer's report on

The Habilitation Thesis

Kinks and oscillons: Dynamics of fields in 1+1 dimensions

by **Filip Blaschke**

The Habilitation thesis of Dr. Blaschke summarizes his recent work on special solutions of 1+1 dimensional scalar field theories, known as kinks and oscillons. These are examples of solitons, solutions with particle like properties, that exist in many scalar field theories and have important applications in many areas of current research in theoretical physics. The thesis consists of three main parts: an introduction to the topic of solitons, a summary of the techniques used in the author's recent work and finally five of the author's most recent research papers on the topic. These papers constitute a small part of Dr. Blaschke's body of scientific work, comprising over 30 papers published in internationals journals of high impact.

The first two chapters of the thesis give a pedagogical introduction to the topic of solitons in 1+1 dimensional relativistic field theories. The first chapter introduces scalar field theories, their symmetries and Noether's theorem in a very accessible way. The second chapter introduces solitons, via the example of a pendulum chain and its field theory limit – the sine-Gordon (sG) model. The basic sG soliton, the kink solution, is described in detail. Kinks in a general scalar field theory are then discussed as well as the concept of BPS solutions. Finally, kinks in models with at least two scalar fields are discussed. The presentation is logical, self-contained and pedagogical and would be easily accessible to a student at the master's level.

After this general introduction, the third and fourth chapter introduces the topics studied in the papers included in the thesis. Chapter three focuses on oscillons, which are somewhat mysterious soliton-like solutions which are long lived despite not being protected by any conserved charge or by topology. These objects are then studied via the decay of an initial Gaussian field profile in various models. An intriguing connection to Q-balls, charged solitons in a complex scalar field theory, which is the topic of one of the author's recent papers, is described.

The fourth chapter is focused on collective coordinate models, reductions of the degrees of freedom adapted to describing soliton dynamics. A case is made for a general purpose "agnostic" collective coordinate model described in two of the papers. It is based on a certain discretization of the scalar field model leading to a mechanical system and dubbed "mechanization". It is shown that this approach can reliably reproduce several features of soliton solutions.

In summary, I found the thesis to be very well written with very few typos. The exposition is clear and logical which makes it enjoyable to read. The work of Dr. Blaschke presented in the thesis constitutes a valuable contribution to the study of solitons in 1+1 dimensional field theories and it fulfills the requirements of a habilitation thesis in the field of theoretical physics.

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Dear Committee Members

Report on Habilitation Thesis of Filip Blaschke
Kinks and Oscillons – Dynamics of fields in 1+1 dimensions

This thesis is based on research work by the author since about 2021, which has appeared in five coauthored papers that are attached to the thesis. There are two main topics. One is the modelling of continuous classical field dynamics using a discrete approximation called "mechanization". This gives new insight into the dynamics of the topologically stable solitons known as kinks. Here the author appears to be the initiator and leader of the research, working with his students. The second topic is the search for and discovery of a relationship between the non-topological particle-like objects in field theory known as oscillons, which are very long-lived but not completely stable, in terms of a more stable type of soliton known as a Q-ball. Here the author has collaborated with a strong research group based in Krakow. I have also collaborated with this group on related problems, and learnt something of Filip Blaschke's research through interesting talks he gave at the annual workshops in Krakow.

The first chapter of the thesis gives an excellent review of classical scalar field theory in general – its Lagrangian, field equation, and the consequences of symmetry and topological structure. The author distinguishes the Noether conservation laws arising from symmetry, that require the field equation to be satisfied, from the topological conservation laws that are even more general, and only depend on certain natural boundary conditions being satisfied. The second chapter reviews the topological kink solutions that occur in a range of nonlinear scalar field theories in one space dimension. When kinks move, and possibly oscillate, they satisfy both Noether and topological conservation laws. There is a brief review of the special reduced equation – the BPS equation – satisfied by a large class of static kinks, including those occurring in multi-field models. These chapters form a valuable set of notes that could be the basis for a one-semester course on nonlinear field theory at Master's or upper undergraduate level. With the addition of a section on oscillons, such notes could bring students more-or-less to the research frontier in classical field theory, particularly field theory in one spatial dimension.

The third chapter presents the novel insight into oscillons. Oscillons have been much studied for about three decades but still present mysteries, and there is no complete theory. Much of what is known is through numerical simulation. Oscillons are periodic and localised excitations of a scalar field (in 1-, 2- or 3-dimensions, although here the focus is on 1-dimension). The oscillon profile is not a simple analytic function, and its exact shape is only known approximately, for example, through a truncated asymptotic expansion. The field profile approaches, far away on both sides, the same vacuum configuration, so there is no topologically non-trivial structure. As a result, oscillons can slowly radiate energy to infinity, and eventually they decay away, but the detail is hard to understand and much research has aimed to clarify and model the slowness of the decay. A fairly recent discovery is that the oscillon amplitude can itself slowly oscillate. This then produces a modulated oscillon, which is quasi-periodic, with two principal frequencies. It still slowly decays.

Here, several of the numerically observed phenomena are understood by viewing the oscillon as the real section of a solution of the field equation for a complex field. The complex field dynamics has a conserved charge Q, and the complex solution is known as a Q-ball (even in 1-dimension). The detailed profile of an oscillon depends on the shape of the field theory potential (its Taylor expansion around the minimum representing the vacuum). Free field theory requires just the quadratic mass term, and exhibits no oscillon. Oscillons require nonlinearity, and a cubic term alone is known to be sufficient. There is therefore a rather canonical oscillon in a theory with cubic potential. What is shown here, remarkably in my view, is that a subtle nonlinear rescaling of variables – the renormalisation group perturbation theory – can accommodate a generic combination of cubic and quartic potential terms, and reduce the oscillon dynamics to a canonical form, extracted from a universal complex equation (eq.(3.46)) which has a Q-ball solution. The derivation of this result has some similarity to the perturbative analysis of purely real, small-amplitude oscillons due to Fodor et al., but is not the same. The key idea is to work perturbatively in the oscillon amplitude, which leads to a sequence of linear equations with sources, and to avoid secular terms that would destroy the oscillon periodicity. This avoidance places constraints on various constants of integration that would otherwise be arbirary. The mechanism that combines the cubic and quartic coefficients is impressive.

The Q-ball equation can be converted, to a high degree of accuracy, to a complex version of the exactly integrable sine-Gordon (sG) equation. This has exact periodic, "breather" solutions, that are a type of oscillon with infinite lifetime. An interesting insight in chapter 3 is that the sG equation is known to have exact multi-breather solutions which are quasi-periodic. An explicit example of a 2-breather is given in eq.(3.61), with the breathers on top of each other and not in relative motion. From this, a real oscillon with modulated amplitude can be obtained. The transformation does involve some approximation, so what emerges is a long-lived modulated oscillon that still decays. This extension of the relationship between a simple sG breather and the simplest oscillon is a highlight of the thesis. This chapter raises the interesting issue whether there is a general theory of multi-breather solutions in the real and complex sG equations. How are they classified, what is their spectral data, and can they be used to obtain more general quasi-periodic oscillons?

The fourth chapter has a quite different approach to scalar kinks and their interactions. The author, with student collaborators, has discretised the dynamics, by approximating the continuous field by its values at a discrete sequence of locations along the spatial axis. Unlike in a lattice discretisation, with an infinite sequence of equally spaced locations, here the locations themselves, as well as the field values at these points, vary dynamically and are not constrained. The motivation is to obtain a good approximation to the dynamics of kinks depending on only a finite, and preferably small number of physical degrees of freedom, different from the more usual approach of collective coordinates. The latter is criticised as depending too much on knowledge of the kink solution itself and of its low-frequency, small amplitude vibrational modes, which are far from universal. The new approach is “agnostic” and should work in the same way for a wide variety of scalar field models. This is a worthy and novel aim. More than one discretisation is suggested, with slightly different treatments of where and how the field potential energy is evaluated. An elegant version does this evaluation half-way between neighbouring locations, leading to a BPS-type discrete kink with definite energy. The price for this is a kind of gauge-invariance, as the kink energy become independent of the distribution of the locations, although the field dynamics still depends on the distribution.

In a bit more detail, Chapter 4 first explores how static kink solutions are approximated, and how the approximation improves as the number of locations N increases. The optimal configurations are far from having equally spaced locations – instead, these locations are concentrated where the second derivative of the continuous kink profile is large. The chapter then moves on to the modelling of kink-antikink collisions in this scheme, something previously studied using collective coordinate methods. The dynamical equations are derived from those of the usual field theory by replacing space derivatives by finite differences. The new dynamics mimics rather well some features of the well-known, complicated kink-antikink scattering dynamics. Depending on initial velocity, sometimes kinks simply annihilate into radiation, but often they bounce, forming a temporary oscillatory field. This can last very long, or there can be a later kink-antikink annihilation after one or more bounces, or the kink and antikink can separate. In some range of velocities, the discrete and continuous dynamics are similar, as the author anticipated, but in others the behaviours disagree.

A curious, and rather unwelcome feature of the discrete scheme is that some locations where the field is evaluated can move off to infinity, and effectively disappear. One can be left with a much simplified kink. with just a single linear connection between the topologically distinct vacua. Other problems arise when two locations of the discrete scheme coalesce. The Riemannian metric in the discrete configuration space becomes singular, and the dynamics breaks down. More than one remedy for this problem is proposed, but these require further exploration. The model does not seem to admit the possibility of a dynamically increasing number of locations, although this could lead to an increasingly refined approximation to the continuum dynamics, analogous to mesh refinement in lattice approximations.

The conclusion seems to be that mechanization, as an approximation to continuous field dynamics, has some success but also has some problems. It seems to me that the most interesting interpretation for the scheme is as a model for a truly discrete dynamical system, with field values attached to a finite number of moving particles, and the dynamics depending both on the particle nearest-neighbour separations and on the local, time-varying differences in field values. Such a model easily extends to more than 1-dimension.

In summary, this thesis, and the attached papers which give more details in several cases, has novel and stimulating ideas concerning kink and oscillon dynamics. It also has numerous valuable figures based on numerical studies. The author has shown considerable research productivity in the past few years, on topics going beyond those he studied before 2020. Most interesting is the combination of multi-breathers of the sG and related Q-ball equations to explain the long-lived modulated oscillons. This could be developed into a general multi-breather theory, not relying on specific 1-breather and 2-breather solutions. Also interesting, and original, is the mechanization of field dynamics – replacing the continuous field by a finite number of field-carrying moving particles. The author, in reviewing earlier work, has an excellent and mature idea of the strengths and weaknesses of various ideas that have been used to understand field dynamics in one spatial dimension. He also makes honest assessments of his own research, and what needs further study.

Although I am generally positive about this thesis, and think it worthy of a habilitation, there are some details which could and should be improved. (In my country, the candidate is usually expected to make minor corrections to a PhD thesis, following a viva, before the PhD is finally approved.) In the present case, I think this thesis should be corrected before it is deposited in a university library or published online.

Some of the corrections are purely typographical, and can be communicated privately after the committee has made its broader assessment. A few I wish to point out (apologies for any errors of mine):

In the Abstract, the author mentions solitons as endemic. One or two examples, particularly of kinks or 1-dimensional oscillons, should be mentioned.

"Per partes" is usually written "integration by parts".

ϕ missing in eq.(1.49).

Lagrangian density is defined after eq.(2.2), but it should be defined earlier, in context of eq.(1.33).

In Fig. 2.9 graph, x-ordering not consistent between left and right.

p.44 Better to say frequency threshold rather than energy threshold.

p.45 and following: sphaleron misspelt, and a sphaleron corresponds to a saddle point, not local minimum.

eq.(3.9) λ is an inverse width.

p.52 Original reference for 2-breather formula should be given.

eq.(3.20) error is probably $O(f^4)$.

eq.(3.61) This is Ψ_{12}.

Some concluding paragraph for chapter 3 would help, along the lines of paper IV, summarising what has been established.

The derivation of eq.(4.15) (in paper I) should be cited.

In paper V, after eq.(5), the lengths in the examples don't all look right (e.g. the ϕ^4 potential has zero third derivative at the maximum).

Yours sincerely

Nicholas Manton FRS
Professor of Mathematical Physics