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July 23, 2010

Report on

The doctoral thesis
ON A SPECTRAL FORMULATION OF QUANTUM MECHANICS
by **RNDr. Jan Kotůlek**

In this thesis, a new formulation of Quantum Mechanics using Spectral Geometry is investigated from various points of view. Section 1 contains mostly historical comments on Quantum Mechanics while in Section 2 the notion of a spectral triple is introduced. In section 3 we are introduced to SQM and its application in non-relativistic theories. In particular, the meaning of the axioms of SQM are nicely elucidated. Finally, in Section 4, the author attempts to apply the SQM formalism to relativistic theories with spin as well as giving historical comments about the invention of the Dirac equation and related topics.

The thesis is written in a clear style in (mostly) understandable English with historical notes and comments throughout the text. This makes it quite enjoyable to read even though I am not able to judge the correctness or importance of the historical notes. The only thing that I would like to comment on is that in the historical parts it seems that the author gets carried away by his love for the topic and is using a language which is somewhat difficult to understand (for instance, what is a "spinose pathway to glory"?).

I have collected a list of questions or comments for the author to address during the defense. They are mostly typos but some of them are suitable for a discussion. I would mainly be interested in hearing questions 6,13,18,20 discussed.

1) It seems to me that the author could have been more clear about when he is working in the Schrödinger or Heisenberg picture. For instance, on page 14 it is said that we are in the Schrödiger picture and then on page 16 (in

Dyson's version of Feynaman's proof) we are suddenly without warning in the Heisenberg picture.

2) In Dyson's version of Feynman's proof there seems to be ordering problems in the Lorentz force equation.

3) To be honest, I do not understand how Dyson's version of Feynman's proof is an inspiration for SQM. That needs to be better explained.

4) In the beginning of page 20, the notation $M_n(\mathbb{C})$ is used before it is defined.

5) In (1.26) and (1.27), what does $\dot{\nabla}$ mean?

6) In section 2.1 we are told that the wave function is a section of a complex line bundle. It would be interesting to hear a discussion on how we are to choose this line bundle. Is it always the trivial bundle or can it be more complicated. For instance, when the base space is S^2, there is an infinite amount of line bundles available.

7) Is it the same n that is used to number the states in the third paragraph on page 24 and in equation (2.2)? It seem to me that they are different but then it would have been safer to use different notation.

8) On the first line of page 25, does the notation x^n mean a real (or complex) value x to the n'th power or does it mean a (infinite dimensional) vector x^n?

9) In the last sentence of section 2.1 what does "violation of $\mathcal{H}_K$" mean?

10) Part (b) of Definition 2.4 is written in such a way that one thinks that one should already know what P_λ is.

11) In Definition 2.6 the author would probably want to call the operator a *raising* operator rather than a rising operator.

12) In Example 2.12 one is led to think that the metric can be arbitrary with signature $(- + +)$ whereas the example is true as stated only if the metric is the Minkowski metric.

13) The author shows that one may define a "complex structure" on the Hilbert space of solutions to the Dirac equation. Why is it advantageous to think of this division as a complex structure rather than as a product structure. To me the notion of product structure seem more natural in this case and it is also more general (no need to have the same number of ± 1 eigenvalues).

14) Just before Definition 3.1, the line bundle E is not defined (see my comment above where the line bundle was specified as the trivial one).

15) In Definition 3.1 (b) $\mathcal{A}_t'$ is not defined. Is it the commutant?

16) In Lemma 3.3 the Hilbert space $\mathcal{H}^\infty$ is suddenly denoted $\mathcal{H}_\infty$.

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17) In eq. (3.6) there is a $\sum_m$ missing.

18) Section 3.3 is in my opinion excellent. It really helps with the intuitive understanding of the meaning of the axioms of SQM. Once the meaning of the axioms are understood one may start questioning them. For instance, is it really necessary that $\ddot{b}$ should be expressable in terms of Newton's law? One could imagine that Newton's law gets "quantum corrections" or that the set of time derivatives closes at a higher level. Also, is it necessary that the Hamiltonian is second order? In quantum field theory we know that the effective Hamiltonian has terms of infinite order in derivatives, the important thing is that the second order term is not zero and then the rest is treated in perturbation theory? Also, in relativistic theories and in theories with gauge symmetry, positivity of the metric is not assured. It would be very interesting to try to apply SQM to some more complicated theories like this.

19) I do not think that a unitary element of some algebra is called "a unitary".

20) Is it not misguided to try to construct a relativistic one particle theory? Since we have the phenomenon of pair production in any relativistic theory, particle number is not conserved and the probabilistic interpretation of the wave function does not (in fact cannot) hold any more.

It is my opinion that this thesis fulfills all the requirements of a PhD thesis in the Czech Republic. After hearing the above comments properly discussed I do not hesitate to recommend that this thesis should be accepted.

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