Report on the Habilitation Thesis in Mathematics. Mathematical Analysis: On Li-Yorke sensivity and other types of chaos in dynamical systems

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In this Habilitation Thesis, the author deals mainly with the notion of *Li-Yorke sensivity* which is a mixture of the notions of *Li-Yorke chaos* and *sensivity on initial conditions*, supposing that the phase space of the dynamical system is a non-empty compact metric space.

The Memoir of the Habilitation is a general view of the implications among some notions of chaos. It is presented under a scheme where it is visual to appreciate the relationships of the considered notions of chaos and some other notions of the topological dynamics in discrete dynamical systems, such as *weak mixing, transitivity, minimality* and *factor map.*

The author considers minimal systems and four conjectures stated by E.Akin and S.Kolyada named by C_1, C_2, C_3 and C_4

(1) C_1: In a minimal system, spatio-temporal chaos is equivalent to Li-Yorke sensivity.
(2) C_2: Every minimal Li-Yorke sensitive system has a non-trivial, weakly mixing factor.
(3) C_3: If $\pi : (X,T) \to (Y,S)$ is a factor map between minimal systems with (Y,S) Li-Yorke sensitive, then (X,T) is Li-Yorke sensitive
(4) C_4: Every minimal system with a weakly factor is Li-Yorke sensitive

Theorem 1 contained in $[M_1]$ disproves C_1 in the sense that there is a minimal dynamical system which is also spatio-temporally chaotic but it is not Li-Yorke sensitive. The system is $Q \times \mathbb{S}$ where Q is the Cantor ternary set

Theorem 2 from $[M_2]$ disproves C_2. The author construct a system which is Li-Yorke sensitive but not weakly mixing. The construction implies the election of skew-product homeomorphisms from a family $\mathcal{F}_2$ of them holding the former property.

Theorem 3 from $[M_2]$ disproves C_3 in the sense that there exists a minimal system which is not Li-Yorke sensitive but having a Li-Yorke sensitive factor. Even more,in fact in the family constructed in $[M_2]$, any system of the form $(Q \times \mathbb{S}, F)$ with $F \in \mathcal{F}_2$ has an almost one-to-one minimal extention (X, t) where T is an homeomorphism but is not Li-Yorke sensitive.

The conjecture C_4 was introduced by Akin and Kolyada. In Theorem 5 from $[M_3]$ is given a partial solution proving that in some cases, a skew-product of product of minimal mixing system and finite phase space is Li-Yorke sensitive. Even more, the system is also Li-Yorke chaotic (see Theorem 6 from $[M_3]$).

Theorem 5 can be generalized to the setting o some types of skew-products of $X \times Y$ where the set Y could be infinite compact.

In the Habilitation Memoir it is presented by three published papers where are contained the former refered results named as (M_1, M_2 and M_3). Such papers are published two of them in *Nonlinearity* and the third in *Journal of Difference Equations and Applications* which are prestigious journals in the settindg of discrete dynamical systems.

In my opinion, the results presented in the Memoir are valuable, interesting, open the possibility of continuing with the research and has scholar sufficient quality. The Memoir is well written and the author proves to possess a sufficient maturity in research in the field of Dynamical Systems and also in Mathematical Analysis.

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