A referee report on the PhD Thesis *Hamiltonian operators and related structures* by Jiřina Vodová-Jahnová

September 3, 2013

Hamiltonian properties of partial differential equations play a crucial role in the theory of infinite-dimensional integrable systems. Though the number of publications in the field is measured by hundreds (if not thousands), there still remains a lot of unsolved problems, both of a general theoretical nature and quite concrete ones. Some of these problems are dealt with in the Thesis under consideration.

The main results by the Author are published in three papers [*Nonlinearity* **24** (2011), **26** (2013), and *Journal of Math. Analysis. Appl.* **(401)**] and exposed in Sections 5–7 of the Thesis. Section 5 concerns with a choice of the Darboux coordinates for the family of the Hamiltonian operators $D_x^2 \circ ((1/u) \circ D_x)^{2n} \circ D_x$ (found by V. Kac with co-authors) and the description of a transformation that transforms this family to $-D_y^{2n+1}$. The main result of Section 6 is a complete and explicit description of conservation laws for the so-called compacton equation $u_t = aD_x^3(u^n) + bD_x(u^n)$. Finally, in Section 7 fifth-order scalar Hamiltonian operators in one independent variable that possess momentum are described. Some interesting examples are considered and an algorithm to find out whether an operator has momentum is described.

All the results are quite interesting. Nevertheless, I have a number of remarks.

1. On p. 5, where the definition of a conservation law is given, the Author demands that $\mathrm{Div}(R)$ must vanish on smooth solutions of the equation at hand. With such a definition, this would mean that any R is a conservation law for equations that possess no smooth solution.

Moreover, to work with this definition efficiently one needs to know all smooth solutions. It would be better to change 'smooth' to 'formal'.

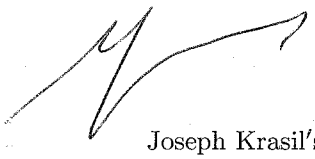
2. The same relates to the definition of symmetries (p. 6).

3. The notion of quasi-constant coefficients (p. 9 and several times further) is not defined in the text.

4. I do not agree that only 'minor technical assumptions' on a bi-Hamiltonian system is needed to ensure complete integrability. One of these assumptions is 1-acyclicity of the Poisson complex and this assumption is neither 'minor' nor 'technical', to my opinion.

5. Something wrong, as I understand, in the formula for ρ_1 in Theorem 16 (p. 20).

In spite of these these remarks, the general impression on the Thesis is quite favorable. The text is written in a clear and strict language. The results obtained are original and new. The Thesis by Jiřina Vodová-Jahnová is on a solid international level and published in good journals. I am sure that the Author deserves the Ph.D. degree in Mathematics.



Joseph Krasil'shchik

July 26, 2013