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Variational sequences in field theory

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Abstract

Variational sequence of the order r introduced by Krupka is defined over a fibered manifold $\pi : Y \to X$, $\dim X = n$, as a quotient sequence of the de Rham sequence of sheaves. In the thesis we pay attention to the 1st-order variational sequence in a field.

We show that there exist isomorphisms between the sheaves of the 1st-order variational sequence and the spaces of differential forms, thus the local representation of the variational sequence by means of forms has been done, explicitly. The quotient morphism which assigns to every q-form ρ (the element of the sheaf of the de Rham sequence) its class of equivalence (the element of the sheaf of the variational sequence) is substituted by the morphism which assigns to every q-form ρ a differential form, representing the class of equivalence of ρ. Further, we provide the explicit description of the morphisms of the 1-st order variational sequence.

Next, the problem of existence of globally defined representatives of the variational sequence in a field theory is solved. Namely, we provide the existence of the globally defined differential forms, which represent the equivalence classes of n-forms, resp. $(n+1)$-forms, resp. $(n+2)$-forms. These representatives are said to be lagrangians, resp. Euler-Lagrange forms, resp. Helmholtz-Sonin forms.

Finally, the n-th (Euler-Lagrange mapping) and $(n+1)$-th (Helmholtz-Sonin mapping) morphisms of the variational sequence of the order one are studied with respect to the calculus of variations. In particular, the problem of local and global

variationality of some specific $(n+1)$-forms on J^3Y, resp. the problem of local and global variational triviality of some special lagrangians of the order two is solved. Thus, the kernel and the image of the Euler-Lagrange mapping of the 1-st order variational sequence in a field is completely described.

References

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