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Report on

The Habilitation thesis
Spektrální geometrie prostoročasu
by Tomáš Kopf

The thesis consists of three papers, all concerning the problem of describing and constructing sensible space-time manifolds in the framework of spectral geometry. The hope is that non-commutative geometry, which most naturally can be introduced through the language of spectral geometry, would provide an alternative way to regulate the notoriously badly behaved Quantum Field Theories of nature, and in particular that it would provide a way to handle the Quantum Field Theory of Gravity, thus giving us a way to attack the most fundamental problems of nature without having to take the route through String Theory. This ambitious program has been very successful in recent years and a lot of progress has been made toward this goal. However, one problem that remains, and one which becomes particularly important if we want to attack questions related to General Relativity, is that so far the models have been formulated in Euclidean signature and the problem of the formulation in physical Lorentzian signature is unsolved.

All three papers in the thesis are concerned with solving this difficult problem. It is clear that they do not completely solve it, but that was not to be expected. They rather provide, as they call it, a "working hypothesis". The basic idea is to consider space-times which can be foliated in to a collection of space-like hyper-surfaces. Then the "standard" procedure can be applied to describe each hyper-surface and the problem becomes the problem of describing how this description (which in some sense *is* the hyper-surface) evolves in time in such a way that the result is a smooth (well, more or less, the question about quantum fluctuations of geometry is an interesting one) space time.

Paper 1: Here the basic ideas of the spectral description of space-time is given and it is suggested that a way to select smooth time evolutions from the set of possible time evolutions is to require that the so called Hadamard states are preserved in time. One is here using the fact that the Hadamard condition is preserved by smooth time evolution in ordinary Quantum Field Theory on curved space and turning this argument around one is hoping that it would give rise to smooth time evolutions in spectral geometry.

Paper 2: The basic axioms defining a spectral quadruple, an extension of the concept of a spectral triple, are given here and it is shown how space-time and its metric can be found given a spectral quadruple. A special case, the symmetric spectral quadruple, is singled out since in that case quite explicit examples can be found. It works as a toy model for the general case.

Paper 3: This paper is concerned with the same issues as in paper 2. The main addition is that an explicit example of a spectral quadruple, 1+1 dimensional de Sitter space, is worked out as an explicit test of the axioms proposed.

The problem addressed in this thesis is interesting and relevant since for spectral geometry to be relevant to physics it needs to be solved one way or another. I believe that the papers represent a contribution toward the solution of this problem. I therefore recommend that this habilitation thesis is accepted.

Sincerely,

Rikard von Unge

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