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Report on the Habilitation Thesis in Mathematics: «Integrability and Geometry» (Submitted by Mr. Hynek Baran, PhD)

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In this Habilitation Thesis, the author considers modern algebro-geometric methods in application to

Non-linear Partial Differential equations and studies their integrability in the framework of the approach and methods developed by A.M. Vinogradov and his school.

The Habilitation Memoir contains:

- 1. A brief introduction and an account of these methods: Jets and geometry of PDE's, linearization, symmetries (local and non-local), differential coverings etc. He also describes few classical paradigms of the PDE integrability and their solutions: recursion, Backlund transformations and Lax-Zakharov-Shabat (or zero curvature) representation.
- The main body of the Memoir is based on 7 author's papers (with various collaborators) where he studies: - in the first part – the integrability of 4-dimensional modified Martínez Alonso– Shabat equation. He has described its recursion operator and an infinite commuting hierarchy of nonlocal symmetries.

- The next part of Thesis covers the results of the candidate about a class 3D Lax-integrable equations which he has called (4E) and which contains four famous various reductions of a 5D equation studied by the author (with I. Krasil'shchik. O. Morozov and P. Vojcák). Mr. H. Baran studies a Lie algebra of local symmetries, differential coverings in the sense of Krasil'shchik and Vinogradov, These coverings provides a complete description of non-local symmetries of these equations as well as a description of their recursion operator action. The class (4E) in its turn admits a 2D reduction whose integrability properties are inherited (probably indirectly) from the latter of 3D prototypes. Namely the Lax-Zakharov-Shabat representations transforms after reductions into the non-linear coverings providing infinite series of non-local conservation laws, while the recursion operators do not preserve under reductions.

- Finally, a different differential-geometric approach to Integrability is discussed in the last part of the HDR Memoir. The systems of Gauss-Mainardi-Codazzi (GMC) and Gauss-Weingarten (GW)

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equations in the 3D Euclidean space are a well-known tool to study properties of CMC (constant mean our vature) surface. The case of non-zero CMC surfaces is specified by the fact that their Gauss maps are no longer holomorphic: they are merely harmonic. The surfaces with integrable systems of GMC-GW are called often «soliton surfaces».

Mr. H. Baran had studied a specific integrable class of surfaces which he calls «surfaces of constant astigmatism». They are described by an integrable PDE in 2D. He (in a collaboration with M. Marvan) studied the surfaces with corresponding integrable Gauss- Weingarten and Gauss-Codazzi -Mainardi equations. It was well-known that the «linear Weingarten surfaces» (= surfaces with a linear relation between Gaussian and Mean curvatures) are integrable. Finkel and Wu had conjectured that this linear relation is the only possible relation between these two curvatures, such that the Weingarten surface is integrable or soliton one.

H. Baran and M. Marvan had disproved the conjecture studying the above-mentioned constant astigmatism equation.

Their result I consider as the most fascinating link between the surface geometry and integrability in this area, classically unknown and geometrically completely unexpected!

My personal impression after few contacts with Mr. Baran are very positives, I was impressed a lot with his maturity in modern computer symbolic computations. It is not a big secret that the numerical simulations have played fundamental rôles in suggesting several possible geometric facts that were eventually proved true.

He (together with M. Marvan) is a creator of the package «Jets» for all simulations and computations with geometry of PDE's, jet spaces, diffieties etc.

All results of his Thesis are correctly published in well-known peer-reviewing journals such that

«J.Phys. A», «Nonlinearity» etc.

Concluding, I think that the HDR Memoir of Mr. Hynek Baran contains important, interesting and sometimes even fascinated results and shows the author's potential as a serious and independent researcher in the area of non-linear PDE's Geometry and symbolic computations for PDE's who has full capacities to be advisor and a mentor of young colleagues.

Professor Vladimir Roubtsov

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