Loughborough University

21 January, 2018

Mathematical Institute in Opava

Report on the PhD thesis by Adam Hlaváč

Constant astigmatism equation and surfaces of constant astigmatism

The thesis is aimed at a detailed investigation of surfaces of constant astigmatism - a remarkable counterpart of constant negative Gaussian curvature surfaces in the Euclidean 3-space. The PDE describing such surfaces is analysed by applying the full machinery of the modern theory of integrable systems including symmetries, (nonlocal) conservation laws, Lax representation, soliton solutions, Bäcklund/reciprocal transformations, and so on.

The thesis starts with a nice historical overview placing the constant astigmatism surfaces in the context of classical differential geometry, followed by the derivation of the constant astigmatism equation (CAE) in the specially adapted coordinates. This preparatory material leads to the main results of the thesis which can be summarised as follows:

1. A superposition formula for the CAE is obtained, based on the analogous result for pseudo-spherical surfaces. This construction leads to a whole variety of solutions to CAE by purely algebraic manipulations.

2. An elegant relation of CAE to equi-areal orthogonal spherical patterns is observed, suggesting potential applications in the context of two-dimensional plasticity.

3. The remarkable 4-parameter class of Lipschitz solutions of CAE is characterised from the symmetry point of view.

4. First-order conservation laws of CAE are explicitly calculated, and the corresponding reciprocal transformations are introduced. These are applied to the known solutions of CAE to obtain new solutions in parametric form.

5. Non-local conservation laws based on the zero curvature representation are explicitly constructed. The structure of the whole hierarchy of conservation laws is investigated, their independence is demonstrated.

6. Multi-soliton solutions of the CAE are constructed based on analogous formulae for sin-Gordon equation. This, however, is only possible in parametric form. Beautiful pictures of the resulting shapes are included.

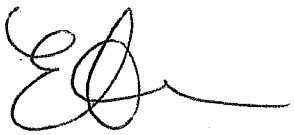
These results fill in a serious gap in the theory of constant astigmatism surfaces which,

despite their geometric elegance and importance, were not sufficiently studied in the literature.

Most of the results of the thesis have been reported at numerous international conferences, and are published in leading academic journals specialising in the areas of differential geometry and integrable systems. The thesis is very well written (in immaculate English) and beautifully illustrated. It clearly demonstrates the candidate's academic maturity and deep expertise in the field.

I give my strongest recommendation for the award of a PhD degree.

Yours sincerely,



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