UNIVERSIDAD
DE MURCIA
DEPARTAMENTO DE MATEMÁTICAS

Professor Jaroslav Smítal
Director of the Institute of Mathematics
Chair of the Habilitation Commission
Opava
Czech Country

Murcia,

October 20, 2003

Dear Professor Smítal,

Enclosed you can find my referee´s report on the Second Doctorate Thesis presented by Marta Štefánková to obtain the degree of "Docent" in Mathematics-Mathematical Analysis.

It only remains to say that it have been a great pleasure for me to do this work and also to give my congratulations to the candidate for the excellent work she has presented.

Sincerely yours

Dr. Francisco Balibrea
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The work presented by RNDr. *Marta Štefánková*, Ph.D. as a second thesis in Mathematical Analysis represents a good contribution in the field of Discrete Dynamical Systems (X, f). In particular, in the knowledge of the chaotic behavior of such systems when X is a general *compact metric space* or when $X = [0,1]$ or $[0,1]^2$.

The work submitted to referee starts disproving a conjecture stated by Agronsky and Ceder saying that a *continuum* $K \subset \mathbb{R}^n$ is an *orbit enclosing* $\omega - \lim$ *it set* if and only if K is *arcwise connected*. The conjecture is disproved simply constructing an example of a map F and looking for a point (x, y) for which is $\omega_F(x, y) \subset \mathbb{R}^2$ orbit enclosing and not arwise connected. Moreover, the map F can be choosen in the class of *triangular maps*, that is

$$F(x, y) = (f(x), g(x, y))$$

Another positive contribution of the author is made in the problem stated in the eighties by several authors, if it is possible or not contruct examples of chaotic maps in the *sense of Li and Yorke* possesing a *measurable scrambled* set of full Lebesgue measure on $[0,1]$. In the eighties and nineties positive answers were given. Now the author proves that similar results can be obtained for *distributionally chaotic* maps (according with the notion introduced by Schweizer and Smtal). In fact she proves that any *bitransitive* continuous map f on $[0,1]$ (f^2 *is transitive)* is conjugate to a map *uniformly distributionally chaotic* almost everywhere and also that for a map f with *positive topological entropy* $(h(f) > 0)$ there is a k such that f^k is *semiconjugate* to a continuous uniformy distributionally chaotic almost everywhere and as a consequence, also chaotic in the sense of Li and Yorke.

A similar result is obtained by the author jointly with J.Smtal using the notion of ω-*chaos* introduced by S.Li. Any transitive continuous map f on $[0,1]$ is *conjugate* to a map g possesing an $\omega -$ *scrambled* set of full Lebesgue measure. It remains open the problem of constructing a continuous map on I^2 with an $\omega -$ *scrambled* set of full planar Lebesgue measure.

In the setting of triangular maps on I^2, jointly with J.Smtal, the author constructs a map of tye 2^∞ having positive topological entropy and holding additionally some properties. In fact this map is distributionally chaotic in a wider sense that what was introduced by Schweizer and Smtal.
She obtains also that for a continuous map on a compact metric space, the two notions of distributional chaos are invariant with respect to topological conjugacy.

Considering now the setting of infinite minimal compact Hausdorff spaces containing a *regular recurrent point,* she proves that each continuous (and hence transitive) map f can be extended to a triangular map F on $X \times I$ such that;

(1) F is also transitive, (2) $h(F) = h(f)$, (3) F has two minimal sets, namely, $X \times \{0\}$ and $X \times \{1\}$, (4) F is distributionally chaotic.

Since the different notions of chaos are not equivalent in the class of continuous maps on compact metric spaces, it is necessary to distinguish among them. When $X = [0, 1]$ then

$$DC = \omega C = PTE \subset CLY$$

where with $DC, \omega C, PTE$ and CLY we denote respectively the set of continuous distributional chaotic,$\omega - chaotic$, positive topological entropy and Li-Yorke chaotic maps.

The author states that in the setting of general compact metric spaces is

$$LYC \supset (DC \cup \omega C \cup PTE)$$

Nevertheless it remains to do a lot of work in this problem in order that this classification must be complete.The construction of counterexamples is the task in this case.

In my own, the author supplies interesting and valuable results in the theory of *chaotic maps on compact metric spaces* and continues the progress made in the last ten years. Besides, the author have stated new problems which will allow her to work in the same line.

The presented papers have been published in good journals in the field and part of the quotations of such papers she presents are relevant.

Therefore it seems to me that the Second Doctorate Thesis of $Marta\ \check{S}tef\acute{a}nkov\acute{a}$ **is excellent.**

Francisco Balibrea