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Dear Professor Englis.

Here is my Referees report concerning Z. Dusek "Homogeneous geodesics in homogeneous manifolds with an invariant affine connection".

The study of Riemannian and pseudo-Riemannian "go" spaces has been a fruitful and important one. One says that a homogeneous manifold (M,g) is a "go" space if every geodesic (possibly after reparametrization) arises in the form t->exp(tx)P for some 1 parameter subgroup of the group of isometries. Any naturally reductive space is such an example, but there are examples which are not naturally reductive. Furthermore, in the higher signature setting, there are examples where the geodesic has to be reparametrized in the form s=e^{kt}. One can also study the purely affine setting and discuss analogous results.

The thesis in question consists of a number of papers. The first is an unpublished and expository account of the field. It is extremely well written and provides an elegant and instructive introduction to the field that puts things into a natural perspective.

The second "New examples of Riemannian g.o. manifolds in dimension 7" was published in Differential Geometry and its Applications and is joint with O. Kowalski and S. Nikcevic. Riemannian g.o. manifolds in dimension 6 have been classified; prior to this paper, there was only a single 7 dimensional example known. In this paper several compact 7 dimensional g.o. spaces are discussed together with their non-compact duals. The authors consider (SO(5)xSO(2))/U(2) or (SO(4,1)xSO(2))/U(2) and construct a family of invariant metrics which are irreducible not naturally reductive Riemannian go spaces. The primary technical tool which is employed is that of a "geodesic graph". The paper is an engaging one – the examples provided are both instructive and elegant.

The third paper "Structure of geodesics in the flag manifold SO(7)/U(3)" discusses a very specific example; it arises in a natural series of flag manifolds and thus is worth investigating as a preliminary start on the family in question; a 2-parameter family of metrics is constructed of degree 4 that are go spaces but not naturally reductive. The calculation is algebraic in nature and rather technical. Nevertheless, the final result is instructive and well presented. Again, the canonical geodesic graph plays a central role in the discussion.

The next paper "Light like homogeneous geodesics and the geodesic Lemma for any signature" turns to the pseudo-Riemannian setting where additional new phenomena appear; in particular, the parametrization of the geodesic in group theoretic terms is not the natural parametrization as a geodesic. The paper is joint with Kowalski and appears in Math. Debrecen. The fundamental equation has to be modified to read <[X,Z]_m,X_m>=k<X_m,Z> -- the constant k is zero in the Riemannian setting and in the pseudo-Riemannian setting relates to the geodesic parametrization. Using the geodesic Lemma, a 3 dimensional unimodular Lorentz group is discussed.

The paper "On 6-dimensional pseudo-Riemannian almost g.o. spaces" returns to the 6 dimensional setting in the indefinite signature context. It is joint work with Kowalski and appeared in J. Geom. Physics. A 6-dimensional Riemannian g.o. spaces is modified to yield 2 pseudo-Riemannian homogeneous spaces with non-compact isotropy group. It is not a go space, but all geodesics are homogeneous up to a set of measure zero. It is a fascinating example in that it exemplifies phenomena which can only appear in the higher signature setting and which are not simply trivial extensions of the positive definite context. Again, the formidable algebraic complexities are handled with skill and grace.

The next paper "Almost go spaces in dimensions 6 and 7" is a single author paper which appeared in Advances in Geometry. It presents in the pseudo-Riemannian setting, appropriate extensions of 6 and 7 dimensional Riemannian examples. Again, the examples are not go spaces but all geodesics are homogeneous up to a set of measure 0. This is typical in this subject – for example there are many complete curvature homogeneous manifolds manifolds which are not homogeneous but which are homogeneous if a set of measure zero is removed. The paper is an important one since it is a single author paper. As always, the computation is lie-algebra theoretic and the combinatorial complexity is formidable.

The paper "On the reparametrization of affine homogeneous geodesics" passes from the pseudo-Riemannian to the affine setting. Examples are presented of affine homogeneous geodesics (i.e. geodesics which arise as a 1-parameter of diffeomorphisms preserving the given connection) where the geodesic parameter is not the parameter arising from the group involved. Again, the paper is a single author paper. The examples appear in low dimension (m=2) which permits a relatively complete treatment. Two different families are presented, each requiring a different system of PDE's.

The paper "The existence of homogeneous geodesics" is again a single author paper in J. Geometry and Physics. Results concerning the existence of homogeneous geodesics in the pseudo-Riemannian setting are derived by passing to the more general affine context.

The final paper "Scalar invariants on special spaces of equiaffine connections" is to appear in J. Lie Theory and is a single author paper is in a quite different context. The focus is not on geodesic theory but rather on constructing scalar invariants in the affine setting – this in contrast to the Riemannian and the pseudo-Riemannian setting is rather difficult to do and the results are novel and somewhat surprising.

Conclusions: The work presented shows a significant level of mathematical breath and development. It starts in the Riemannian setting, branches out to the pseudo-Riemannian setting, includes the affine setting, and ends with scalar invariants. It is both single author and coauthored – this demonstrates the author is both capable of working in a group and also capable of working alone. The work presented is important, deep, and shows a breath of understanding and uses different techniques and has clearly had a significant impact on the field. Although the techniques used are often very combinatorial, they are handled well and the exposition is useful in this regard. It is therefore a pleasure to recommend the thesis for acceptance.

Respectfully submitted

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