Report on the Habilitation Thesis:

Homogeneous geodesics in homogeneous manifolds with an invariant affine connection

A central topic in Riemannian geometry is the study of geodesics. Their behavior is determined by the Riemannian structure but they also influence it as shown, for instance by the Hopf-Rinow Theorem. It is clear that the geodesics of some special classes of Riemannian manifolds (two-point homogeneous, symmetric, homogeneous, etc.) will exhibit some specific properties and conversely, one may try to characterize some special classes of manifolds in terms of the behavior of their geodesics.

A Riemannian homogeneous manifold is called a g.o.-space if every geodesic is the orbit of a 1-parameter group of isometries. Naturally reductive homogeneous spaces are go-spaces and they are the only examples in dimension ≤ 5. The existence of other examples, motivating the investigation of this class of manifolds, which started with the work of Vinberg and Konstant and further developed by Kowalski and his collaborators.

Clearly one may also consider the pseudo-Riemannian case (where difficulties are expecting in dealing with null geodesics) and the affine setting. This is the framework of the Thesis under consideration, which consists in some contributions by the Author .

The Thesis consists of 9 chapters, the first one being an introduction to the topic and the contents of the subsequent chapters.

Six dimensional g.o.-spaces which are not naturally reductive were classified by Kowalski and Vanhecke and a not naturally reductive example 7-dimensional example was constructed by Gordon. This was the motivation for the investigation presented in Chapter 2, where new examples of compact irreducible g.o.-spaces which are not naturally reductive were constructed. Chapter 3 discusses a special example: the flag manifold $SO(7)/U(3)$, whose associated geodesic graph has degree 4.

Starting at Chapter 4 the discussion moves into the pseudo-Riemannian setting. Being an essential tool in the Riemannian case, a pseudo-Riemannian generalization of the geodesic lemma is approached in this chapter. This generalization presents some difficulties coming from the existence of null geodesics and their parametrizations. Chapter 5 focuses in the analysis of some pseudo-Riemannian analogs of an example by Kaplan. Although the examples are just modifications of Kaplan's example, they exhibit a completely different geometric behavior, thus illuminating on the difficulties of the pseudo-Riemannian case. Here the analysis of almost g.o.-spaces (homogeneous spaces whose geodesics are homogeneous up to a set of measure zero) is started and further developed in Chapter 6, where the six-dimensional analysis was extended to dimension 7.

The affine case is considered in the last three chapters, where similar questions are considered. The last chapter is devoted to investigate scalar invariants in affine geometry (a rather different but related topic). This is a very open problem which certainly deserves further attention.

The whole Thesis is a consisted piece of work, part done by the Author alone, part in collaboration (mainly with O. Kowalski). Therefore showing the Author capability of working both along and in a group. The topics and the techniques used are important and highly non-trivial. The results here presented have had an important impact and are basic references for anyone studying analogous problems.

Summarizing all the above, I am very pleased to RECOMMEND the acceptance of this Habilitation Thesis.