CZECH TECHNICAL UNIVERSITY IN PRAGUE
FACULTY OF NUCLEAR SCIENCES
AND PHYSICAL ENGINEERING

Břehová 7 115 19 Praha 1

Report on the habilitation thesis

Symmetries and conservation laws of differential equations

submitted by Roman O. Popovych

The thesis presents important results in the modern theory of differential equations. It contains five research papers written by the candidate either alone or with one coauthor, preceded by an introduction to the field of research and the results. The papers deal with three different topics:

- Conditional symmetries and nonclassical reductions of differential equations:

  [**T1**] Kunzinger M. and Popovych R.O., Singular reduction operators in two dimensions, J. Phys. A: Math. Theor. 41 (2008), 505201, 24 pp.

  [**T2**] Popovych R.O., Reduction operators of linear second-order parabolic equations, J. Phys. A: Math. Theor. 41 (2008), 185202, 31 pp.

  [**T3**] Kunzinger M. and Popovych R.O., Generalized conditional symmetries of evolution equations, J. Math. Anal. Appl. 379 (2011), 444-460.

- Potential conservations laws:

  [**T4**] Kunzinger M. and Popovych R.O., Potential conservation laws, J. Math. Phys. 49 (2008), 103506, 34 pp.

- Invariant parameterization schemes:

  [**T5**] Popovych R.O. and Bihlo A., Symmetry preserving parameterization schemes, J. Math. Phys. 53 (2010), 073102, 36 pp.

In the first paper [T1] singular reduction operators of conditional symmetry of partial differential equations (PDEs) in two independent variables are introduced and applied to evolution equations and nonlinear wave equations. Next, it is studied which properties of a PDE and a subset of its reduction operators lead to a so-called "no-go" situation when a single symmetry determining equation is equivalent to the initial PDE.

In the paper [T2] the reduction operators of $(1 + 1)$-dimensional second-order linear parabolic PDEs and all their possible reductions to ODEs are described. The main result of the paper is a chain of no-go" statements on reduction operators. These statements

show that the application of conventional methods to solving the determining equations for coefficients of such operators cannot lead to reduction operators giving solutions of the given PDEs. Nevertheless, it is demonstrated that using certain empiric approaches one can find reduction operators leading to new exact solutions of the considered PDEs.

In the paper [T3] the authors study the relationship of generalized conditional symmetries of evolution equations to the formal compatibility of systems of differential equations. This leads to a no-go theorem on determining equations for operators of generalized conditional symmetry.

In the paper [T4] the authors study several possible constructions (Abelian and general coverings, and standard potentials) of a chain of conservation laws for potential systems for the given system of PDEs. They prove that potential conservation laws have characteristics depending only on local variables if and only if they are induced by local conservation laws. Therefore, characteristics of pure potential conservation laws have to essentially depend on potential variables.

In the last paper [T5] parameterization schemes are studied and a novel approach towards them is proposed. This problem arises in averaging of nonlinear differential equations, e.g. Navier–Stokes equation, for its numerical solution. In the process additional undetermined terms arise. These terms must hence be re-expressed in a physically reasonable way to be included in the averaged equations in a process called parameterization. The authors propose to use as a guiding principle the requirement that the parameterization scheme should preserve the group of Lie point symmetries of the original equations (or its physically relevant subgroup), propose several methods accomplishing it and demonstrate their approach on the averaged vorticity equation.

The papers present interesting and relevant results and are written in a mathematically rigorous way, thus showing the candidate's mathematical expertise. All of them were published in international journals well recognized by the scientific community.

I would like to suggest several topics for discussion during the habilitation presentation:

1. Given that the candidate has published (many) more papers on all three research topics discussed in the thesis, I would be curious to know what was the guiding principle in selecting the presented papers? E.g., does he consider these papers to have made a particular impact on the field or present the most original ideas?

2. Since the presented papers were published in 2008-2011 and, with the exception of Ref. [48] dating to 2017, all references cited in the introductory text go back to 2014 or earlier, I would appreciate a brief comment by the candidate on recent

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development in his field of research and applications of his results presented in the thesis, both by himself and his collaborators, as well as other research groups.

3. At least conceptually, the idea of symmetry preserving parameterization schemes reminds me of the research programme on symmetry preserving discretizations of V. Dorodnitsyn, D. Levi, P. Winternitz. Do these two approaches have something more in common, beyond the basic idea of preserving the relevant algebra of symmetries in the process?

To sum up, the thesis conforms to the requirements of Act No. 111/1998 Coll., on Higher Education Institutions, and clearly demonstrates the candidate's ability of establishing his own scientific agenda and accomplishing important and consequential results. I recommend without any hesitation the thesis to be accepted and the habilitation procedure to proceed.

In Prague on May 18, 2019

doc. Ing. Libor Šnobl, Ph.D.