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Report on the Memoir entitled: **Properties of chaotic discrete dynamical systems** presented as Habilitation Thesis by Zdenek Kocan.

In the first part of the Memoir, the author presents a short introduction of what is the main topic treated on it and some comments concerning the contain of the papers selected to this concurs. The second part is a presentation of some reprints where can be seen most of the research made by the author.

After introducing the subject, the author makes a recall of the standard terminology and notation in discrete dynamical systems and then he concentrates in three topics: recurrence,different notions of chaos and other related notions in the setting of discrete dynamical systems.

In the section devoted to recurrence, he recall the definitions of recurrent, uniformly recurrent and regularly recurrent points and gives how the same notions are called in some literature. He brings to our attention four notions of chaos: *chaos in the sense of Li and Yorke*, *ω-chaos*, *topological chaos* and *distributional chaos*, with their variants depending the number of the scrambled that can appear, like $LY_2-chaos$, $LY_\infty-chaos$, $LY_u-chaos$, $\omega_2-chaos$, $\omega_\infty-chaos$ or $\omega_u - chaos$. In the case of *distributional chaos*, the three versions on it.

In other notions, he brings other definitions connected with the chaotic behavior like existence of *homoclinic trajectories* or *eventually periodic homoclinic trajectories*; existence of *horseshoes*, *endpoint intersection horseshoes* or *finite intersection horseshoes*.

In the setting of general *compact metric spaces*, there are some implications among the above notions of chaos, but unfortunately not too much when we face a very general setting. To obtain more results it is necessary to concentrate in compact metric spaces with additionally properties. In this sense, in last years has been extended the interest in the research of dynamical systems in continua, particulary in *trees*, *graphs*, *dendrites*, *dendroids*, etc, that are currently considered as a important part of the research in Topology. The author does it for the problem of the implications of different notions of chaos. Of particular difficulty is to obtain results in dynamcial systems on dendrites. Some of them are in the quoted papers. I find of particular interest the following:

There is a sequence of ω-limit sets $(\omega_k)_{k=1}^{\infty}$ *of a continuous self-map f of a dendrite such that* $\omega_k \subset \omega k+1$, *for every positive integer k, but there is no ω-limit set of f containing every* ω_k

This means that the set of ω-limit sets is not closed in the Hausdorff metric. It is known that the above result is true for continuous interval maps and also for some types of triangular maps on the square.

In the last quoted paper (currently submitted), are considered such connections among the different kind of chaos, particularly topological chaos, existence of different types of homoclinic trajectories, three level of Li-Yorke chaos, three level of ω-chaos and distributional chaos of type 1 and surveyed what is the

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situation in intervals, trees and graphs. Also is is stated the same problems for dendrites. The implications not solved are formulated as open problems.

An important part of the research of the author is devoted to *triangular* or *skew product* maps on the square ($[0,1]^2 = I^2 = Q$)

$$F(x,y) = (f(x), g(x,y)), \quad (x,y) \in Q$$

where f and g are continuous maps. The research has been concentrated in getting equivalent conditions to the fact that $h(F) = 0$ where $h(F)$ denotes the *topological entropy* of F. Originally, such research was initiated by Sharkovskii, followed by S.Kolyada. They stated the problem of seeing if the equivalent properties (a lot of them) to have zero topological entropy of interval maps are true or not in the triangular setting, that is, they proposed to research in what extend from a dynamical point of view Most of the results of the author go in such direction even including more topics than initially, like different types of *distributional chaos.* Also the scheme of implications has been complete by the research of the Group of Dynamical Systems from Opava.

There are 23 stated properties to see if they are equivalent or not and currently, only 9 of them are left to be proved or disproved. Such open problem are formulated properly by the author,

In my opinion, the results obtained and presented in the Memoir by Z. Kocan are relevant for the theory of Dynamical Systems (skew product maps and chaotic systems) and means a great effort by him. Additionally his results have been published in journals of a good scientific level.

Therefore I think the Habilitation Thesis presented by the candidate has a good scientific level and therefore **deserves** to b considered as of good level.

Additionally I would recommend him to the Habilitation Commission to get the promotion to *Docent* in the area of Mathematics - Mathematical Analysis.

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