Report on the habilitation thesis

"Chaos and omega-limit sets in one-dimensional dynamical systems",

by RNDr. Michal Málek, Ph.D., for promotion to the rank of "Docent" in the area of Mathematics - Mathematical Analysis.

Since the appearance (or rediscovery) in the seventies of some very influential papers by Sharkovsky and Li and Yorke, interval topological dynamics has garnered a lot of attention. Dynamics in more complicated one-dimensional spaces (trees, graphs, dendrites) has been investigated as well. It must be emphasized that this is not just generalization for the sake of it. While interval dynamics are pretty well known by now, we are far from understanding discrete dynamics in dimension 2. Yet, dendrites often appear as Julia sets (that is, the sets where complicated behaviour lies in) in complex dynamics, see for instance G. Levin and S. van Strien, *Local connectivity of the Julia set of real polynomials*, Ann. of Math. (2) **147** (1998), 471–541. Thus, these more general one-dimensional spaces could be seen, in a sense, as a bridge between dimensions 1 and 2. The involvement of some very prestigious mathematicians in this particular field (Alsedà, Blokh, Downarowicz, Franks, Llibre, Misiurewicz, Snoha...) is further proof of its relevance.

In his habilitation thesis, Dr. Málek investigates some fundamental dynamical notions (mainly omega-limit sets and chaos) in this setting. Among many other results, he characterizes omega-limit sets in graphs, shows that a dendrite map need not have a maximal omega-limit set, and unifies and investigates in deep basic sets in graphs. On the other hand, he studies the relations among positive topological entropy, homoclinic orbits, horseshoes, periodicity, basic sets and distributional chaos, mainly in graphs but also, partially, in dendrites.

The thesis comprises eight published papers (five of them in journals included in the Journal Citation Reports lists) and a preprint (submitted to a journal included in the JCR lists as well). These journals implement serious peer review processes, which guarantees the technical correctness of the results in the thesis. The employed tools are often non-trivial. They are mainly topological in nature, of course, but I was favourably surprised to see that ergodic theory is cleverly used in the paper published in *Dynamical Systems.*

In short, the results in this thesis are new, interesting and non-trivial and, in this reviewer's opinion, accredit the expertise and skills of Dr. Málek

on the subject. Thus I think that Dr. Michal Málek satisfies the standard requirements for habilitation and recommend his promotion to the rank of "Docent".

Signed: Prof. Víctor Jiménez López
Universidad de Murcia, Spain