An opponent report
on the Habilitation Thesis
Invariants in Relativity and Gauge Theory
by Igor Khavkine
submitted to
Mathematical Institute in Opava

The Thesis under consideration consists of six papers by Igor Khavkine (one of them in collaboration with two other co-authors) four of which where published in various journals and two are accessible in `arXiv`. A general overview in concise form is given in the Introduction to the Thesis. To this end, I believe that it makes no sense to discuss minor details of the exposition (such as typos, not perfect formulations sometimes, etc.). So, I shall try to express my general opinion on the main results and their presentation.

All the papers combined in the collection are united by a common idea: application of the geometry of partial differential equations and related cohomology theories to modern gauge-invariant field theories. Actually, such an approach goes back to works of M. Henneaux with co-authors and theoretically in many respects is based on the pioneer work by A.M. Vinogradov, *The spectral sequence, Lagrangian formalism and conservation laws*, J. Math. Anal. Appl., **100** (1984), pp. 1–129 (unfortunately the author does not cite this paper).

The first paper of the collection, *Local and gauge invariant observables in gravity*, Classical and Quantum Gravity, **32** (2015) 185019, deals with relations between the theory of differential invariants and various gauge theories as they are understood in the classical and quantum field theory. The basic theory considered in the paper is general relativity. The main problem here lies in the well-known fact that general relativity does not admit local invariant observables. A natural generalization given by Dr. Khavkine allows to obtain a large class of physically meaningful observables that are invariant w.r.t. diffeomorphisms, local or global.

In *The Calabi complex and Killing sheaf cohomology* (Journal of Geometry and Physics **113** (2017) 131–169), the structure of the Calabi complex is considered from a new standpoint. Explicit compuations based on representation theory for the Lie group $GL(n)$ are given together with some tools to study cohomology of the complex in terms of locally constant sheaves. These tools may be adapted to linearized gravity other gauge theories. The author gives explicit formulas for the differential operators in the Calabi complex and discusses generalized Poincaré duality, etc.

The third paper presented by the author, Cohomology with causally restricted supports (Annales Henri Poincaré, **17** (2016) 3577) addresses to the following problem. De Rham cohomology with spacelike compact and timelike compact supports are important for understanding the structure of classical and quantum Maxwell theory on curved space-times, different cohomologies play a similar role in other gauge theories. A method to compute these causally restricted cohomologies is described. The method is applied to the de Rham complex and to the Calabi complex, which is important in linearized gravity on constant curvature backgrounds (see above).

The fourth and fifth papers (*IDEAL characterization of isometry classes of FLRW and inflationary spacetimes*, Class. Quantum Grav. **35** (2018) 035013, in collaboration with G. Canepa and C. Dappiaggi, and *IDEAL characterization of higher dimensional spherically symmetric black holes*, unpublished, see `arXiv:1807.09699 [gr-qc]`) deal with the so-called IDEAL (Intrinsic, Deductive, Explicit, ALgorithmic) characterization of a reference spacetime in general relativity. The first IDEAL characterization of the cosmological Friedmann–Lemaître–Robertson–Walker space-times is given (with and without a dynamical scalar field). Regular geometries are considered, which uniformly satisfy certain identities or inequalities. Also, a first IDEAL characterization of generalized Schwarzschild–Tangherlini (FLRW) space-times are given, which consist of Λ-vacuum extensions of higher dimensional spherically symmetric black holes.

Finally, the last paper, *Compatibility complexes of overdetermined PDEs of finite type, with applications to the Killing equation* `arXiv:1805.03751 [gr-qc]`, the problem of gauge-equivalence $h_{\mu\nu} \sim h_{\mu\nu} + K_{\mu\nu}[v]$ for linearized metrics modulo the image the Killing operator K is considered. To this end, the compatibility complex for the latter is constructed. The results are applied to the cosmological FLRW space-times in any dimension and to a generalization of the Schwarzschild–Tangherlini black hole spacetimes, also in any dimension.

All the main results presented in the above mentioned papers are, as I can judge, new and interesting for all mathematicians and mathematical physicists who work in classical and quantum field theories and applications of geometrical theory of PDEs to physics. The material is exposed in a clear, self-contained way with reasonable rigor. I would also like to add that I attended a number of lectures delivered by Dr. Igor Khavkine at several international conferences and his oral exposition is equally clear and understandable. The results obtained by the author are of a good international level and play an important role in the modern mathematical gauge theories. I am sure that I. Khavkine deserves the degree of *Docent* at Mathematical Institute of Silesian University in Opava.

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