A REVIEWER REPORT ON ARTUR SERYEYEV'S HABILITATION THESIS
SYMMETRIES, RECURSION OPERATOR AND HAMILTONIAN STRUCTURES OF INTEGRABLE SYSTEMS

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Though the thesis text consists of several independent works by the Author published at different time, all these works are unified by main common topics and some additional subjects closely related to these topics.

To my opinion, the core topic of the Thesis relates to nonlocal constructions that arise in geometry of integrable systems. These problem are extremely important in many respects. One of them is expressed in the title of the fourth paper included in the Thesis: *Why nonlocal recursion operators produce local symmetries?* The author gives a sort of an answer (maybe not an exhaustive one, but I really doubt that an exhaustive explanation exists) that explains locality of hierarchies generated by the so-called weakly nonlocal operators. By the way, another question arises: why almost all recursion operator we find in applications are weakly nonlocal ones?

The lack of exhaustive solutions to "nonlocal" questions is (partially, at least) explains by the fact that the settings generated by nonlocal variables are immense and usually non well defined. Even in such a construction like the universal Abelian covering, canonically defined for any equation, it is impossible to deal efficiently with all nonlocal variables. The first paper of the Thesis (*On Recursion Operators and Nonlocal Symmetries of Evolution Equations*) sheds light upon the geometry of this covering. An old conjecture by N. Khor'hova is clarified and proved: under certain assumptions, nonlocal symmetries generated by recursion operators contain non-localities associated with conservation laws of the initial equation only.

It is well known that recursion operators for integrable systems are closely related to bi-Hamiltonian properties of these systems (via the Magri scheme), and in the last paper (*A Simple Method of Making a Hamiltonian system into a bi-Hamiltonian One*) we find a procedure to construct bi-Hamiltonian structures for systems of hydrodynamical type. Though not directly related to nonlocal problems, essentially the results belong to the same area. In this respect I have just a minor remark: the Author cites my paper on Poisson structures, ref. [20], but a more adequate reference would be to Lecture Notes in Math. **1334**, Springer-Verlag, 1988. But of course, this is not really essential.

In the second paper included in the Thesis (*A Remark on Nonlocal Symmetries of the Calogero–Degasperis–Ibragimov–Shabat Equation*) one finds one of few examples where the infinite symmetry algebra is completely described.

Finally, in *On Sufficient Conditions of Locality of Symmetries of Evolution equations* the notion of a nondegenerate weakly diagonizable system is introduced and for such systems locality of symmetry hierarchies is proved. I think, that this part is the least interesting one in the Thesis. The only application discussed in the text is the Harry Dim equation (a scalar one, by the way), for which locality can be proved in a simpler manner.

On the whole, I firmly believe that the Habilitation Thesis *Symmetries, Recursion Operator and Hamiltonian Structures of Integrable Systems* by Artur Seryeyev is a high-level scientific research in the field of nonlinear integrable systems satisfying all international standards. The results are new, interesting and important and the Author by all means deserves the positions he seeks for.

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