Report

On Habilitation Thesis titled "On some special types of solutions of differential equations" by Karel Hasík

This thesis is a concise exposition on a range of mathematical results obtained by the author in the past 15 years. It is a self-contained presentation with all necessary definitions and preliminaries given and all the principal results stated. All the related publications by the author are attached and they form the second part of the thesis. Therefore, the interested reader can obtained full mathematical details and a comprehensive picture of the entire research work. I presume the way the thesis is compiled is a standard one for mathematical sciences at universities of Czech Republic. I will briefly reflect below on the significance and value of this work, the way I view it (without repeating major mathematical statements and details of the thesis).

The mathematical work consists of two major and weakly related parts:

(I) Existence, uniqueness, and stability of limit cycles in a two-dimensional system of ordinary differential equations (ODEs);
(II) Existence and properties of travelling waves in a class of partial differential equations (PDEs) with delay.

The first part deals with the so-called generalized Gause type model described by a system of ODEs in the (x, y) plane of the form:

$$x' = x\,g(x) - y\,p(x), \quad y' = y\,[q(x) - \gamma], \tag{1}$$

where γ is a constant, and g, p, q are continuous real-valued functions subject to certain additional assumptions. An important motivation for consideration of such systems comes from mathematical modeling of biological systems, such as predator-prey interaction systems, and has a long history of research studies going back for almost a century.

In this work several sufficient conditions are derived for the existence of a unique cycle in system (1). It is also proved that the cycle is globally asymptotically stable (with the asymptotic phase). They are stated as six theorems coming from three publications by the author. These theorems generalize some of the previously known results as well as they establish several new criteria for the existence, uniqueness, and stability. They represent, in my view, an interesting and important contribution by the author to a particular research direction relatively well studied by others. It is not only that he is able to obtain new results complementing and extending some of the previous work in this direction, but he also brings his research to the level where some open questions and new challenging problems arise. Thus, creating a basis for further extensions and future exploration. The results of this part are published in three well-known and recognized international journals.

The second part of the thesis deals with properties of traveling waves in a parabolic type PDE with time delay (the so-called KPP-Fisher equation; the KPP stands for three mathematicians, Kolmogorov, Petrovsky and Piscounov, who initiated an early study of related equations; Fisher introduced the logistic term $u(1-u)$ into it). The equation has the following form:

$$u_t(t,x) = \Delta u(t,x) + u(t,x)(1 - u(t-\tau,x)), \quad x \in \mathbb{R}^m, \tag{2}$$

where $u \geq 0$ and $\tau > 0$ is the delay.

The traveling waves to equation (2) are its solutions of the form $u(t,x) = \phi(\nu \cdot x + ct), |\nu| = 1$, with the profile function ϕ satisfying the following delay differential equation

$$\phi''(t) - c\phi'(t) + \phi(t)(1 - \phi(t-h)), \quad h = c\tau, \ t \in \mathbb{R}, \tag{3}$$

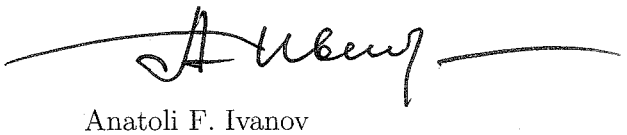
with $\phi(t) \geq 0 \ \forall t \in \mathbb{R}$ and $\lim_{t\to-\infty} \phi(t) = 0$.

A sequence of results is established for the existence (or non-existence), shape, and further properties of traveling waves in equation (2). In particular, parametric subdomains are explicitly indicated in the parameter space (τ, c) where traveling waves are monotone or they are oscillatory (non-monotone). In some cases the uniqueness of the waves is also established (up to time-translation). The results are derived by involving several approaches and sophisticated techniques to study heteroclinic connections and their properties of delay differential equation (3).

The second part is a solid mathematical work that assumes a prominent place within a large number of related results obtained by others in the past 20 years or so. It naturally complements and extends some of the prior results as well as establishes its own important contribution into this field. Besides, it poses a range of open problems and conjectures, therefore paving directions of future research for the author and his students and collaborators.

The results presented in the thesis are published in five prestigious international mathematical journals, thus partially implying their high theoretical level and significant value. Some of the journals are in the very top tire for the field of differential equations and dynamical systems, such as Proceedings of the American Mathematical Society, Journal of Mathematical Analysis and Applications, and Discrete and Continuous Dynamical Systems.

In summary, I find the whole set of mathematical results of this thesis to be an important and valuable contribution to the theory of differential equations, and I think the author well deserves to be promoted to the rank of Docent.



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