# Report on the Dissertation Thesis: Various approaches to the blowing-up orbits technique

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In the Memoir under revision, the author presents two papers, [1] and [2]. Comments, remarks and possible future working research on the subjects contained in the papers, are also made.

In the whole Memoir, there are two links between the papers. The first is that in both, the main results are the construction of counterexamples disproving some conjectures and proving others. The technique used for the counterexamples are is that of blowing up orbits of the considered systems.

In [1], the author faces the relationship between Li-Yorke chaos and positive topological entropy in the setting of non-autonomous dynamical systems. In autonomous systems such relationship has been a problem considered for various authors in different papers. Before it the relationship between Li-Yorke chaos and topological entropy was considered for general compact metric spaces and particularly for discrete dynamical systems on the real interval. For continuous maps on compact metric spaces, was shown that positive topological entropy implies Li-Yorke chaos but the opposite implication is not true. Later, was proved that for maps on the real interval, if there exists a sequence of positive integers such that its topological sequence entropy is positive, then this fact is equivalent to the existence of Li-Yorke chaos.

With the above perspective in view, the author deals with non-autonomous systems where the sequence of maps defining them are uniformly convergents to a funtion, always in the interval case. In fact he proves that contrary to what has been comment above, in non-autonomous case, positive topological sequence entropy and existence of Li-Yorke chaos are not equivalent and this is proved trough the contruction of the announced counterexamples.

In [2] the author, jointly with Sergei Trofimchuk, applies a technique of blowing-up introduced by Hric and Jger, to give an affirmative answer to the problem stated some years ago, on the existence or not of non-invertible minimal circle-fibered self-map on the Klein bottle. This result allows him to give a simpler construction of a non-invertible minimal self-map on the two-dimensional torus.

The solved problems belongs to the class of problem stated to understand the

structure of minimal systems when the phase space of the discrete dynamical system is a manifold of low dimension, and the map defining the system is continuous.

Once more, the author uses a particular blowing-up technique for the construction and it allows to construct some maps on the Klein bottle and in the two torus which are the candidates of the solution. The rest (which is not easy) is to test that the maps are not invertible but are minimal.

In my opinion, the results presented in the Memoir by the author are valuable, interesting and open the possibility of continuing the research. The Memoir is well written and the author proves his maturity sufficiently in the research in mathematics.

I recommend the Comitee to accept the presentation of this Memoir as a Doctoral Thesis and give to it the maximum score.

# References


[1] Jakub Sotola, *Relationship between Li-Yorke chaos and positive topological sequence entropy in non-autonomous dynamical systems*, Discrete and Continuous Dynamical Systemss **38(10)** (2018), 5119-5128.

[2] Janus Sotola and Sergei Trofimchuk, *Construction of minimal non-invertible skew-product maps on 2-manifolds*, Proceedings of the American Mathematical Society, **144(2)** (2015), 723-732.



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