Referee's report on the Habilitation Thesis

Integrability and Geometry

by Hynek Baran

To whom it may concern

The Habilitation Thesis of Hynek Baran is devoted to a geometric study of the integrability properties of interesting non-trivial Partial Differential Equations (PDEs). The results of H. Baran were published in several papers on high-level scientific journals; they are listed in the Preface of the thesis and in this report they will be referred to by means of their corresponding label in the Preface (a Roman numeral).

The first two chapters of the thesis introduce standard material in the field of Geometry of PDEs, following the standard reference [1]).

In Chapter 3 the paper I is discussed. Here the results consist in a new Lax pair for the modified Martínez Alonso – Shabat equation, and a new hierarchy of nonlocal symmetries of the equation. The interesting property of this hierarchy is that it is made of commuting symmetries of the covering of the equation by its Lax pair. This is an uncommon fact, as usually it is only possible to find shadows of symmetries in such a situation.

In Chapter 4, four equations admitting Lax pairs are considered: the universal hierarchy equation, the $3D$-rdDym equation, the Veronese web equation and the Pavlov (or Mikhalev) equation. These are all integrable nonlinear scalar PDEs with 3 independent variables and one unknown function. As in Chapter 3, the main object of research are hierarchies of commuting nonlocal symmetries, that are constructed for the above equations. The results in Chapter 4 come from the papers II, III.

In Chapter 5, the above four equations are studied under the viewpoint of 2-dimensional reductions. In the paper V, the complete list of 2-dimensional symmetry reductions of the above four equations; the analysis gives rise to a list of 32 equations, 16 of which can be solved explicitly, 5 are linearizable, one reduces to the Riccati equation and the remaining 10 are nontrivial. Two equations of the latter list can be transformed to the Liouville and Gibbons–Tsarev equations respectively, thus leaving a list of 8 equations. While 5 equations out of the list have an infinite-dimensional Lie algebra of contact symmetries, 3 have finite-dimensional symmetry algebras. The three equations have been investigated in the paper IV. In this paper, the Authors find that the equations admit infinite sequences of nonlocal conservation laws; however, they also prove that the recursion operators of the original 3-dimensional equations are not preserved in the reduction process.

The last two papers (VI and VII) deal with integrable equations for classes of surfaces in the Euclidean 3-space (this is Chapter 6). In particular in VI there is a classification of Weingarten surfaces for which the corresponding Gauss–Codazzi–Mainardi equations are integrable in the sense that there exists a zero-curvature representation. There are remarkable connections of the results in this paper with the classical literature (Beltrami, Bianchi, Darboux) as well ascompletely new mathematical facts, like the governing equation that is solvable in terms of elliptic integrals. In VII a class of integrable Weingarten surfaces is discovered. This is governed by the nonlinear PDE of constant astigmatism surfaces. The discovery disproves a conjecture that was independently formulated by Finkel and Wu on the fact that integrable Weingarten surfaces were restricted to be either minimal or parallel to surfaces of constant Gaussian curvature.

The scientific production of H. Baran is wide and of high quality. He proved that he masters completely the theory and the calculation of symmetry and conservation laws. In particular, the calculations that H. Baran performed during his scientific activity are absolutely non-trivial and quite uncommon, even between specialists, also revealing good programming abilities.

Indeed, most of the research activity of H. Baran was in the rather unexplored area of multidimensional integrable PDEs, a subject that is absolutely worth of investigation. The fact that few research works can be counted in such a fascinating topic is, in my opinion, due to the fact that experiments are of the highest computational difficulty, and most people do not have the mastery of computer algebra systems that is required to be effective. Hynek Baran certainly has such a mastery.

In conclusion, I do recommend H. Baran for the habilitation to the degree of "Docent".

Best regards,

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References

[1] A. V. Bocharov, V. N. Chetverikov, S. V. Duzhin, N. G. Khor'kova, I. S. Krasil'shchik, A. V. Samokhin, Yu. N. Torkhov, A. M. Verbovetsky and A. M. Vinogradov: Symmetries and Conservation Laws for Differential Equations of Mathematical Physics, I. S. Krasil'shchik and A. M. Vinogradov eds., Translations of Math. Monographs **182**, Amer. Math. Soc. (1999).