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Report on Dr. Hynek Baran's Habilitation Thesis "Integrability and Geometry"

The habilitation thesis of Dr. Hynek Baran presents important and interesting results in a modern and intensively developing field of mathematics -- the geometric theory of differential equations. The thesis is comprised of an introduction, which contains a brief description of the necessary concepts and methods, and seven research papers written by the candidate either alone or in co-authorship. The papers can be split into two sets depending on their topic.

Most of the papers, [I–V], are devoted to extended symmetry analysis of multidimensional integrable equations and their reductions. This analysis includes Lie and contact symmetries, generalized and nonlocal symmetries, cosymmetries, local and nonlocal conservation laws, recursion operators and differential coverings. Although the theory of two-dimensional integrable models was well established and developed in the last decades, the phenomenon of multidimensional integrability is still not sufficiently understood in spite of intensive research in this topic and thus deserves further comprehensive consideration.

The class of equations considered in the paper [I] consists of the four-dimensional modified Martínez Alonso–Shabat equations, which are parameterized by a nonzero real constant. For each of these equations, a Lax pair with non-removable constant parameter has been known, but a recursion operator and an infinite hierarchy of nonlocal symmetries are constructed in the paper [I]. There are only few examples in the literature, where infinite-dimensional algebras of complete nonlocal symmetries rather than their shadows are found for multidimensional integrable partial differential equations, especially in dimensions greater than three, and the paper [I] presents one of such examples.

The subjects of the papers [II–V] are even more closely related to each other. All these papers deal with so-called submodels of the equation (4.7) with five independent variables, which is a particular case of the Manakov–Santini equation. This is clearly visualized by the diagram in Figure 1. Among codimension-one Lie reductions of the equation (4.7), there are the four-dimensional rdDym equation (4.6) and the four-dimensional Pavlov equation (4.5). Further codimension-one Lie reductions of the equations (4.5) and (4.6) in particular lead to the universal hierarchy equation (4.1), the three-dimensional rdDym equation (4.2), the Veronese web equation (4.3) and the Pavlov equation (4.4). These four three-dimensional equations, jointly called the 4E equations in the thesis, are thus codimension-two Lie reductions of the initial equation (4.7). All the above equations are Lax-integrable. For each of the 4E equations (more specifically, for the rdDym equation (4.2) in [III] and for the other 4E equations in [II]), the candidate construct two infinite hierarchies of two-component nonlocal conservation laws, which correspond to nonnegative and nonpositive powers of the spectral parameter and lead to two, positive and negative, infinite-dimensional coverings, respectively. The algebras of nonlocal symmetries associated to these coverings are found, and the action of recursion operators on shadows of nonlocal symmetries is analyzed. The maximal Lie invariance algebras of the 4E equations, the structure of these algebras and all possible inequivalent codimension-one Lie reductions of each of the 4E equations were computed in [V]. Only eight of the obtained reduced equations with two independent variables are of interest for further consideration. In [IV], contact symmetries of these eight equations are computed. The contact-symmetry algebras of five of them are infinite-dimensional. For the other three equations, (5.1)–(5.3), whose contact-symmetry algebras are finite-dimensional, it is analyzed how the integrability properties of the initial three-dimensional equations (4.1), (4.2) and (4.4) behave under the corresponding reductions. More specifically, the Lax representations for equations (4.1), (4.2) and (4.4) reduce to differential coverings for the equations (5.1)–(5.3), respectively. For each of the reduced coverings, an infinite series of conservation laws is constructed and the nontriviality of these conservation laws is proved. Local symmetries and cosymmetries of the reduced equations are described and the corresponding conservation laws are presented. It is also proved that the equations (5.1), (5.2) and (5.3) are pairwise inequivalent with respect to contact transformations.

In total, the papers [II–V] can be considered as a fruitful realization of the extended Ovsiannikov's program 'Submodels'. The initial version of this program suggested by Ovsiannikov in the 1990s includes, in the case of one dependent variable, only computing all possible inequivalent Lie reductions, finding Lie symmetries of reduced systems and further Lie reductions using hidden Lie symmetries. In the papers [II–V], the candidate goes much beyond the scope of the initial Ovsiannikov's program.

It is well known that the classical geometry of immersed surfaces in the Euclidean space is closely related to the modern theory of integrable systems. Studying integrable Weingarten surfaces, Dr. Baran jointly with Dr. Marvan obtained interesting results concerning this relation, which are collected in papers [VI, VII]. The integrability of linear Weingarten surfaces is commonly known for a long time. Drs. Baran and Marvan were able to find a new class of integrable Weingarten surfaces, which they called *surfaces of constant astigmatism*. It is shown in [VII] that the associated nonlinear partial differential equation (6.11), called the constant astigmatism equation, possesses a zero curvature representation (6.12) with a nonremovable parameter and a third-order symmetry (6.14), which were missed in the previous papers on this equation, as well as a recursion operator.

It should be emphasized that Dr. Baran is the principal developer of the well-known package `Jets` for `Maple`, whose creation was initiated by Dr. Marvan. In the thesis, Dr. Baran only mentions his authorship in developing this package. In my opinion, at present the package `Jets` is the most powerful and advanced tool for symbolic computations related to the geometric theory of differential equations. My students and collaborators and personally I use `Jets` in our research. I greatly appreciate the impact of Dr. Baran to this package.

The publication list reflects Dr. Baran's extensive research collaboration with his colleagues from the Silesian University in Opava and with international experts in the field and, as a co-author of a number of research ar1icles, he has published in top-ranked peer-reviewed journals.

All in all, the seven papers collected in the thesis represent an impressive and substantial contribution to the research literature in the field laying in the intersection of geometry, integrability theory and symmetry analysis of differential equations. The awarding of the degree "Docent" to Dr. Hynek Baran is well deserved and has my complete support.

Sincerely yours,
Roman Popovych