

Report on doctoral thesis
“Hamiltonian operators and related structures”
by Jiřina Vodová-Jahnová

The presented doctoral thesis concerns the investigation of the Hamiltonian operators and conserved quantities for some class of nonlinear partial differential equations. The equations in question, known as Hamiltonian evolution equations, are well known to play an important role in mathematical physics and have been extensively studied in the last 40 years. The Hamiltonian operators play the central role in the integrability theory, and various classes of such operators are being classified with respect to the changes of variables.

The doctoral thesis of Vodová-Jahnová is based on three papers, in which she is the only author, published in good international journals. The first chapter is an introduction where the author gave the basic definitions and theorems. The said papers constitute the next three chapters.

The first paper, “*The Darboux coordinates for a new family of Hamiltonian operators and linearization of associated evolution equations*,” is published in *Nonlinearity* 24 (2011), no. 9, p. 2569. It is known that it is often possible to find new variables in which a Hamiltonian operator takes a simple form but finding an explicit transformation to such variables is quite a challenging problem. The main result of the paper consists in finding a transformation which simultaneously turns the recently discovered Hamiltonian operators $\partial_x^2((1/u)\partial_x)^{2N}\partial_x$ into the operators with constant coefficients for all $N \geq 1$.

The second paper, “*A complete list of conservation laws for non-integrable compacton equations of $K(m, m)$ type*,” is published in *Nonlinearity* 26 (2013), no. 3, p. 757. In this paper for the so-called $K(m, m)$ -type equation $u_t = a(u^m)_{xxx} + b(u^m)_x$, where a, b, m are constants, a complete set of local conservation laws is found for $m \neq -2, -1/2, 0, 1$. The author first proved several no-go theorems using which the explicit form of local conservation laws was obtained. This is a worthy result.

The third paper, “*Low-order Hamiltonian operators having momentum*,” is published in *J. Math. Anal. Appl.* 401 (2013), no. 2, p.724. Here the problem of classification of Hamiltonian operators possessing momentum is considered. The momentum of a given Hamiltonian operator Γ is, by definition, a functional Ω such that $\Gamma(\delta\Omega) = u_x$. The concept of momentum is useful, for example, for averaging the systems that are Hamiltonian with respect to Γ . The main result of the paper is a complete description of local fifth-order Hamiltonian operators in one dependent and one independent variable that possess momentum.

There are a few minor typos in the thesis, for example, in the definition of formal symmetry at page 6 of the introduction one should have D_F instead of D_P .

In summary, I find the results presented in the thesis to be new, interesting and correct, and I recommend to award the Ph.D. degree to Jiřina Vodová-Jahnová.

Wrocław 22.07.2013 Prof. dr hab. Ziemowit Popowicz

