

**Report of the habilitation thesis**  
**“Conjugacy equation in dimension one and its applications**  
**in iteration theory”**  
**by Krzysztof Ciepliński**

The thesis concerns mainly the problems of conjugacy of families of self-maps of the unit circle and of intervals. It is based on six published papers, three of them are joint ones with M. C. Zdun:

- [1] K. Ciepliński and M. C. Zdun, *On semi-conjugacy equation for homeomorphisms of the circle*, in: *Functional equations-results and advances*, 135-158, Kluwer Acad. Publ., Dordrecht, 2002.
- [2] K. Ciepliński and M. C. Zdun, *On a system of Schröder equation on the circle*, *Internat. J. Bifur. Chaos Appl. Sci. Engrg.* **13** (2003), 1883-1888.
- [3] K. Ciepliński, *Topological conjugacy of disjoint flows on the circle*, *Bull. Korean Math. Soc.* **39** (2002), 333-346.
- [4] K. Ciepliński, *On conditions guaranteeing that mappings are elements of iteration groups*, *Appl. Math. Lett.* **24** (2011), 1415-1418.
- [5] K. Ciepliński, *Schröder equation and commuting functions on the circle*, *J. Math. Anal. Appl.* **342** (2008), 394-397.
- [6] K. Ciepliński and M. C. Zdun, *On uniqueness of conjugacy of continuous and piecewise monotone functions*, *Fixed Point Theory Appl.* 2009, Art. ID 230414, 11pp.

The thesis consists of three chapters followed by references and reprints of the papers. The first chapter – Introduction – consists of the list of publications forming the thesis and a very brief survey of topics and main results obtained there. The second chapter – Preliminaries – contains notation, definitions and some facts concerning maps of the circle (e.g., lifts, rotation numbers and limit sets) and iteration groups (e.g., disjointness, continuity and measurability of iteration groups). In the third chapter there is a description of main results contained in the papers forming the thesis.

In the paper [1] the authors study when two families  $\{F_t, t \in M\}$ ,  $\{G_t, t \in M\}$  of pairwise commuting orientation-preserving homeomorphisms of the circle such that for at least one parameter  $t_0 \in M$ ,  $G_{t_0}$  has no periodic point are conjugate, i.e., they are looking for continuous solutions of the system of functional equations  $\Phi \circ F_t = G_t \circ \Phi$ .

In [2] the authors deal with the system of Schröder equations  $\Phi(F_t(z)) = c(t)\Phi(z)$ , where  $c: M \rightarrow S^1$  and  $F_t, t \in M$  are continuous maps of the circle. They show that under some further conditions, for every integer  $l$  there is a unique solution  $(\Phi, c)$  of this system with  $\Phi$  continuous of degree  $l$  with  $\Phi(1) = 1$ . The particular case  $\Phi(F(z)) = s\Phi(z)$  (i.e.,  $\text{card } M=1$ ) is also considered in this paper.

Paper [3] concerns the problem of topological conjugacy of disjoint iteration groups  $\{F_t, t \in M\}$  and  $\{G_t, t \in M\}$  of the circle. The author studies three subclasses of these groups (namely dense, non-dense and discrete) and he gives, for each class particularly, a necessary and sufficient condition for topological conjugacy of two groups. He also determines, under some additional assumptions, all homeomorphisms  $\Phi$  of the circle satisfying the system  $\Phi \circ F_t = G_t \circ \Phi$ .

In [4] the author continues the study of iteration group on an open interval initiated by J. Matkowski and proves that if a map  $G$  of an open interval is continuous at least at one point and commutes with two elements  $F_a$  and  $F_b$  of a suitable iteration group  $\mathcal{F} = \{F_t, t \in M\}$  and quotient  $a/b$  is not rational, then  $G$  is an element of  $\mathcal{F}$ . In [5] an analogous problem is discussed for circle maps.

Paper [6] is devoted to the study of equation  $\Phi \circ F = G \circ \Phi$  in the case when  $I, J$  are closed intervals and  $F$  (resp.  $G$ ) is a continuous piecewise monotone self-map of  $I$  (resp.  $J$ ). The authors find some properties of  $F$  and  $G$  that ensure the existence and uniqueness of the solution  $\Phi: I \rightarrow J$  and also ensure that this solution is homeomorphic.

The thesis, as well as the particular papers, have very good logical structure, they are well treated, the problems studied are interesting and the results are of good quality.

**Conclusions.** Summarizing the above, it is a pleasure to recommend the habilitation thesis “Conjugacy equation in dimension one and its applications in iteration theory” for acceptance and to promote Mgr. Krzysztof Ciepliński, Ph.D. to “docent”.

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