UNIVERSIDAD
DE MURCIA
DEPARTAMENTO DE MATEMÁTICAS

Prof. RNDr. Jaroslav Smítal, DrSc.
Chair of the Habilitation Commission

Murcia, April 6, 2005

Dear Professor Smítal,

Attached to this letter you can find my report on the Habilitation Thesis presented to the Habilitation Commission by

RNDr. Roman Hric, PhD.

Sincerely yours,

Dr. Francisco Balibrea
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Report on the Memoir entitled: **Topological Entropy and Zeta Functions in Dynamical Systems** presented as Habilitation Thesis by Roman Hric.

In the first part of the Memoir, the author presents some published results which complete a previous result from Franzová and Smítal of 1991(F-Z result) concerning the characterization of L-Y chaotic interval maps (chaotic in the sense of Li and Yorke) using the tool of the sequence topological entropy.

The result reads: *A map $f \in C(I,I)$ is L-Y chaotic if and only if there is an increasing sequence t of nonnegative integers such that the entropy of f with respect to t is zero ($h_t(f) = 0$)*

The author proves that given an increased sequence of positive integers $t = (t_i)_{i=0}^{\infty}$, there exists a L-Y chaotic interval map $g \in C(I,I)$ such that $h_t = 0$, which means that it is not possible using a universal sequence for the F-Z result.

The F-Z result is extended to maps $f \in C(S^1, S^1)$ and also to maps $f \in C(X,X)$ where X is a compact metric space containing an homeomorphic image of an interval. The same result is obtained if X is a Cantor set.

In the setting of L-Y chaotic circle maps, similarly to interval maps they prove that there is no universal sequence for the F-Z result.

In the second part of the Memoir, the author presents the results he has obtained jointly with João F. Alves and José Sousa Ramos on the problem of counting periodic orbits in Dynamical Systems. Such a problem is of interest not only in a general setting, but also in Ergodic Theory since in most cases, the invariant measures considered concentrate on periodic orbits. The adequate tool to deal with such problem is given by the dynamical zeta function introduced in 1965 by Artin and Mazur $\zeta_f(z)$. Such thechnique was used by Milnor and Thurston in their celebrated paper on piecewise monotone maps introducing the Milnor-Thurston dynamical zeta function $\zeta_f^{MT}(z)$. The authors extend the theory to maps on graphs which are of interest in some problems on Topological Dynamics, introducing the *negative dynamical zeta function* $\zeta_f^-(z)$ and proving that for piecewise monotone graph maps f with finitely many stable periodic points, $\zeta_f(z)$ is rational if and only if $\zeta_f^-(z)$ is rational.

Using dynamical zeta functions, they treat with the *Entropy Conjecture* stated by Shub in 1974 which establishes that topological entropy for a smooth map on a compact differentiable manifold is bounded from bellow by the logarithm of the spectral radius of a induced map in the full homology group associated to the original map *homological entropy*(S-conjecture). In 1975, Manning proved the S-conjecture for the first homology group instead of the full one.

Following arguments contained in Manning's paper, the S-conjecture can be also proved in the setting of piecewise monotone graph maps. Moreover, they prove that the topological entropy is exactly equal to the maximum between its homological entropy and the entropy given by the growth of its periodic points of negative type.

In my opinion, the results obtained and presented in the Memoir by R.Hric are relevant for the theory of Dynamical Systems and means a great effort by his own. Moreover, he has been able to publish them in good journals from the impact index point of view.

Therefore I think the candidate **deserves** to succeed on the opinion of the quality of the Habilitation Thesis he presents.

Additionally I would recommend him to the Habilitation Commission in getting the promotion to *Docent* in the area of Mathematics - Mathematical Analysis.